



## Four-jet production in the $k_t$ -factorisation

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### Abstract

We discuss the single-parton and double-parton scattering (SPS or DPS) effects in four-jet production at the LHC. The calculations of both single-parton and double-parton scattering components are done in the high-energy (or  $k_T$ )-factorization approach. Here we follow our recent developments of relevant methods and tools. The calculations are performed for kinematical situations relevant for two experimental measurements (ATLAS and CMS) at the LHC. We compare our results to those reported by the ATLAS and CMS collaborations for different sets of kinematical cuts. A special attention is given to the optimization of kinematical conditions in order to enhance the relative contribution of DPS in four-jet sample. Several differential distributions are calculated and carefully discussed in the context of recent and future searches for DPS effects at the LHC. The dependences of the relative DPS amount is studied as a function of rapidity of jets, rapidity distance, and various azimuthal correlations between jets. The regions with an enhanced DPS contribution are identified.

# 1 Introduction

So far, complete four-jet production via single-parton scattering (SPS) was discussed only within collinear factorization. Results up to next-to-leading (NLO) precision can be found in [1, 2]. Recently we discussed for the first time production of four jets within high-energy ( $k_T$ -)factorization (HEF) approach with  $2 \rightarrow 4$  subprocesses with two off-shell partons [3].

Four-jet production seems a natural case to look for hard double-parton scattering (DPS) effects (see e.g. Ref. [4] and references therein). Some time ago we analyzed how to find optimal conditions for the observation and exploration of DPS effects in four-jet production [5]. In this analysis only the leading-order (LO) collinear approach was applied both to single and double-parton scattering mechanisms.

Very recently, we have performed for the first time a calculation of four-jet production for both single-parton and double-parton mechanism within  $k_T$ -factorization [3]. It was shown that the effective inclusion of higher-order effects leads to a substantial damping of the double-scattering contribution with respect to the SPS one, especially for symmetric (identical) cuts on the transverse momenta of all jets.

So far, most practical calculations of DPS contributions were performed within the so-called factorized ansatz. In this approach, the cross section for DPS is a product of the corresponding cross sections for single-parton scatterings (SPS). This is a phenomenologically motivated approximation which is not well under control yet. A better formalism exists in principle, but predictions are not easy, as they require unknown input(s), e.g. double-parton distributions that should contain informations about space-configuration, spin, colour or flavour correlations between the two partons in one hadron [6]. These objects are explored to a far lesser extent than the standard single PDFs. However, the factorized model seems to be a reasonable tool to collect empirical facts to draw useful conclusions about possible identification of the DPS effects in several processes.

As discussed in Ref. [5], jets with low cuts on the transverse momenta and a large rapidity separation seem more promising in exploring DPS effects in four-jet production. In the following we shall show our recent results for SPS and DPS calculations obtained for first time in  $k_T$ -factorization approach and concentrate on the study of optimal observables to pin down DPS contributions.

## 2 A sketch of the theoretical formalism

The theoretical formalism used to obtain the following predictions was discussed in detail in [3]. All details related to the scattering amplitudes with off-shell initial state partons as well as with the Transverse Momentum Dependent or unintegrated parton distribution functions (TMDs) can be found in our original paper.

Here we only very briefly recall the basic high-energy (or  $k_T$ -)factorization (HEF)

formula for the calculation of the inclusive partonic four-jet cross section:

$$\begin{aligned} \sigma_{4-jets}^B &= \sum_{i,j} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} d^2k_{T1} d^2k_{T2} \mathcal{F}_i(x_1, k_{T1}, \mu_F) \mathcal{F}_j(x_2, k_{T2}, \mu_F) \\ &\times \frac{1}{2\hat{s}} \prod_{l=i}^4 \frac{d^3k_l}{(2\pi)^3 2E_l} \Theta_{4-jet} (2\pi)^4 \delta \left( x_1 P_1 + x_2 P_2 + \vec{k}_{T1} + \vec{k}_{T2} - \sum_{l=1}^4 k_l \right) \overline{|\mathcal{M}(i^*, j^* \rightarrow 4 \text{ part.})|^2}. \end{aligned} \quad (2.1)$$

Above  $\mathcal{F}_i(x_k, k_{Tk}, \mu_F)$  is the TMD for a given parton type,  $x_k$  are the longitudinal momentum fractions,  $\mu_F$  is a factorization scale,  $\vec{k}_{Tk}$  the parton's transverse momenta.  $\mathcal{M}(i^*, j^* \rightarrow 4 \text{ part.})$  is the gauge invariant matrix element for  $2 \rightarrow 4$  particle scattering with two initial off-shell partons. They are evaluated numerically with the help of the AVHLIB [7] Monte Carlo library. In the calculation, the scales are set to  $\mu_F = \mu_R = \frac{\hat{H}_T}{2} = \frac{1}{2} \sum_{l=1}^4 k_T^l$ <sup>1</sup>.

The so-called pocket formula for DPS cross sections (for a four-parton final state) reads:

$$\frac{d\sigma_{4-jet, DPS}^B}{d\xi_1 d\xi_2} = \frac{m}{\sigma_{eff}} \sum_{i_1, j_1, k_1, l_1; i_2, j_2, k_2, l_2} \frac{d\sigma^B(i_1 j_1 \rightarrow k_1 l_1)}{d\xi_1} \frac{d\sigma^B(i_2 j_2 \rightarrow k_2 l_2)}{d\xi_2}, \quad (2.2)$$

where the  $\sigma(ab \rightarrow cd)$  cross sections are obtained by restricting (2.1) to a single channel and the symmetry factor  $m$  is 1/2 if the two hard scatterings are identical, to avoid double counting. Finally,  $\xi_1$  and  $\xi_2$  stand for generic kinematical variables for the first and second scattering, respectively. The effective cross section  $\sigma_{eff}$  can be interpreted as a measure of correlation in the transverse plane of the two partons inside the hadrons, whereas the possible longitudinal correlations are usually neglected. In the numerical calculations here we use  $\sigma_{eff} = 15 \text{ mb}$  that is a typical value known from the world systematics [8].

### 3 Selected results

First we show some selected examples of the results of the  $k_T$ -factorization calculation in Figs. 1 and 2. In this calculations we used the KMR unintegrated parton distributions. The prediction is consistent with the ATLAS data for all the  $p_T$  distributions.

Not only transverse momentum dependence is interesting. The CMS collaboration extracted for instance a more complicated observables [9]. One of them, which involves all four jets in the final state, is the  $\Delta S$  variable, defined in Ref. [9] as the angle between pairs of the harder and the softer jets,

$$\Delta S = \arccos \left( \frac{\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}}) \cdot \vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})}{|\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}})| \cdot |\vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})|} \right), \quad (3.1)$$

<sup>1</sup>We use the  $\hat{H}_T$  notation to refer to the energies of the final state partons, not jets, despite this is obviously the same in a LO analysis.

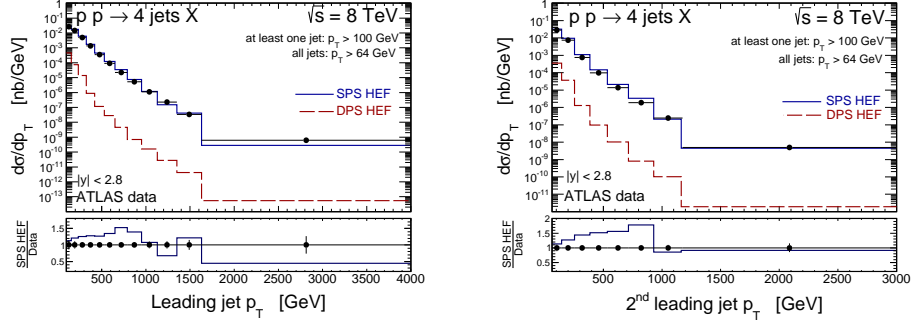


Figure 1:  $k_T$ -factorization prediction of the differential cross sections w.r.t. the transverse momenta of the first two leading jets compared to the ATLAS data [11]. The LO calculation describes the data pretty well in this hard regime in which MPIs are irrelevant. In addition we show the ratio of the SPS HEF result to the ATLAS data.

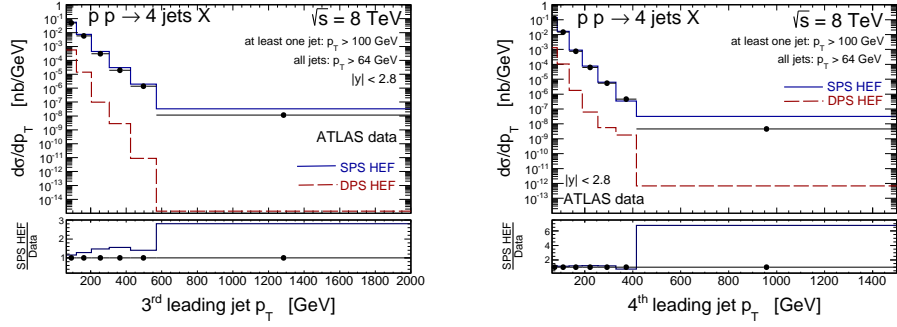


Figure 2:  $k_T$ -factorization approach prediction of the differential cross sections w.r.t. the transverse momenta of the 3rd and 4th leading jets compared to the ATLAS data [11]. The LO calculation describes the data pretty well in this hard regime in which MPIs are irrelevant. In addition we show the ratio of the SPS HEF result to the ATLAS data.

where  $\vec{p}_T(j_i, j_k)$  stands for the sum of the transverse momenta of the two jets in arguments.

In Fig. 3 we present our HEF prediction for the normalized to unity distribution in the  $\Delta S$  variable. Our HEF result approximately agrees with the experimental  $\Delta S$  distribution. In contrast, the LO collinear approach leads to  $\Delta S = 0$ , i.e. a Kronecker-delta peak at  $\Delta S = 0$  for the distribution in  $\Delta S$ .

Now we wish to show a comparison of our numerical predictions with existing experimental data for relatively low cuts on jet transverse momenta. In this context, the CMS experimental multi-jet analysis [9] is the most relevant as it uses sufficiently soft cuts on the jet transverse momenta. The cuts are in this case  $|p_T| > 50$  GeV for the two hardest jets and  $|p_T| > 20$  GeV for the third and fourth ones; the

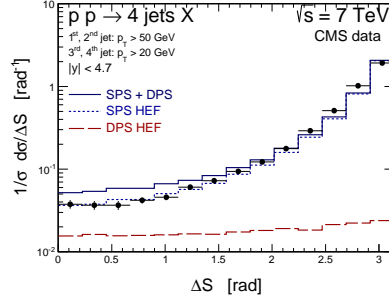


Figure 3: Comparison of the HEF predictions to the CMS data for  $\Delta S$  spectrum.

rapidity region is defined by  $|\eta| < 4.7$  and the constraint on the jet cone radius parameter is  $\Delta R > 0.5$ . The overall situation is shown in Fig. 4, where we plot rapidity distributions for leading and subleading jets ordered by their  $p_T$ 's.

The  $k_T$ -factorization approach includes higher-order corrections through the resummation in the TMDs. However, within this framework fixed-order loop effects are not taken into account. Therefore, we allow for a  $K$ -factor for the calculation of the SPS component. The NLO  $K$ -factors are known to be smaller than unity for 3- and 4-jet production in the collinear approximation case [1]. To describe the CMS data, we also need  $K$ -factors smaller than unity for the SPS contributions, as expected. In contrast to the 4-jet case, the NLO predictions for the 2-jet inclusive cross section are further away from the measured value than the LO ones [1]. The 2-jet  $K$ -factor is known to be about 1.2, and it enters squared in the case of the DPS calculations. However, in our calculations we ignored the relatively small  $K$ -factors for the DPS contribution.

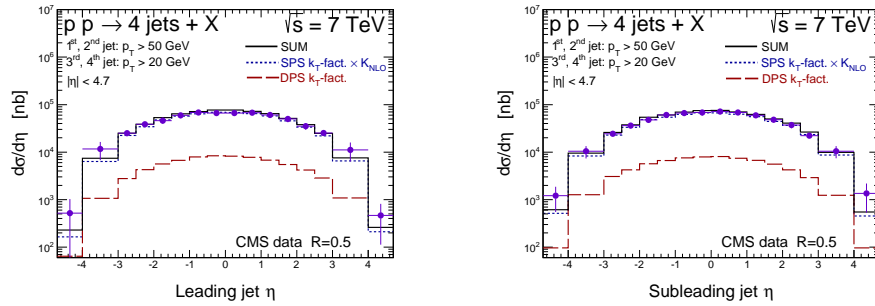


Figure 4: Rapidity distribution of the leading and subleading jets. The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line.

In Refs. [4, 5] we introduced a set of observables that we find particularly convenient to identify DPS effects in four-jet production. Here we present results for

completely symmetric cuts,  $p_T > 20$  GeV, for all the four leading jets. The cuts on rapidity and jet radius parameter are the same as for the CMS case. In Fig. 5 we show our predictions for the rapidity distributions. In contrast to the previous case (Fig. 4), where harder cuts on the two hardest jets were used, the shapes of the SPS and DPS rapidity distributions are rather similar. There is only a small relative enhancement of the DPS contribution for larger jet rapidities.

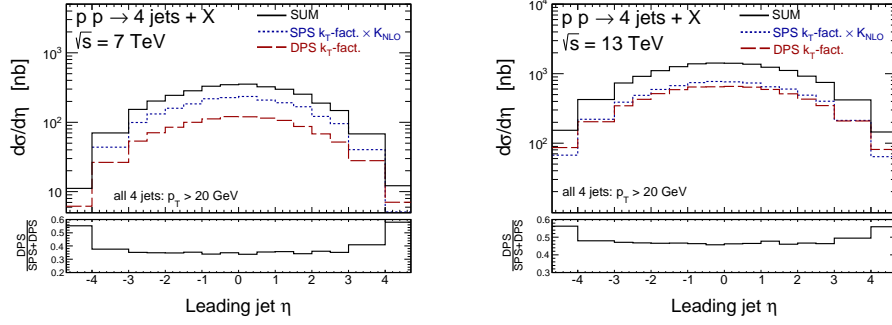


Figure 5: Rapidity distribution of leading jet for  $\sqrt{s} = 7$  TeV (left column) and  $\sqrt{s} = 13$  TeV (right column) for the symmetric cuts. The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.

As it was proposed first in Ref. [10] in the context of Mueller-Navelet jet production, and then repeated in Ref. [5] for four-jet studies in the LO collinear approach, there are two potentially useful observables for DPS effects, such as the maximum rapidity distance

$$\Delta Y \equiv \max_{\substack{i,j \in \{1,2,3,4\} \\ i \neq j}} |\eta_i - \eta_j| \quad (3.2)$$

and the azimuthal correlations between the jets which are most remote in rapidity

$$\varphi_{jj} \equiv |\varphi_i - \varphi_j|, \quad \text{for } |\eta_i - \eta_j| = \Delta Y. \quad (3.3)$$

One can see in Fig. 6 that the relative DPS contribution increases with  $\Delta Y$  which, for the CMS collaboration is up to 9.4. At  $\sqrt{s} = 13$  TeV the DPS component dominates over the SPS contribution for  $\Delta Y > 6$ . A potential failure of the SPS contribution to describe such a plot in this region would be a signal of the presence of a sizable DPS contribution.

Figure 7 shows azimuthal correlations between the jets most remote in rapidity. While at  $\sqrt{s} = 7$  TeV the SPS contribution is always larger than the DPS one, at  $\sqrt{s} = 13$  TeV the DPS component dominates over the SPS contribution for  $\varphi_{jj} < \pi/2$ .

We also find that another variable, introduced in the high transverse momenta analysis of four jets production discussed in Ref. [11], can be very interesting for the examination of the DPS effects:

$$\Delta\varphi_{3j}^{\min} \equiv \min_{\substack{i,j,k \in \{1,2,3,4\} \\ i \neq j \neq k}} (|\varphi_i - \varphi_j| + |\varphi_j - \varphi_k|). \quad (3.4)$$

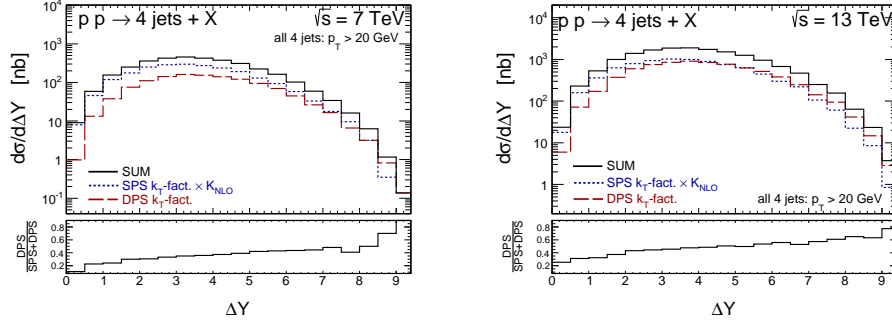


Figure 6: Distribution in rapidity distance between the most remote jets for the symmetric cut with  $p_T > 20$  GeV for  $\sqrt{s} = 7$  TeV (left) and  $\sqrt{s} = 13$  TeV (right). The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.

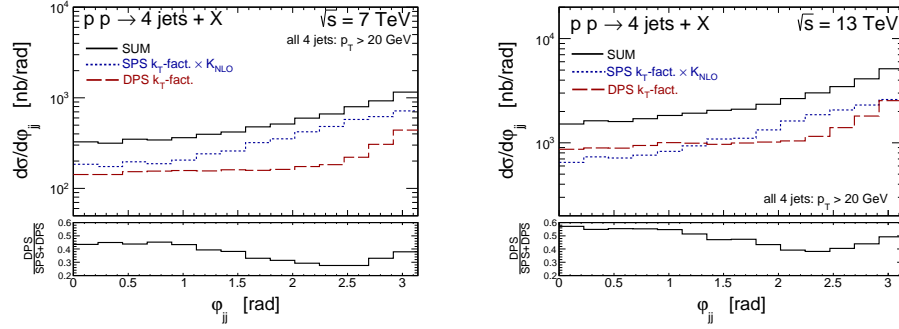


Figure 7: Distribution in relative azimuthal angle between the most remote jets for the symmetric cut with  $p_T > 20$  GeV for  $\sqrt{s} = 7$  TeV (left) and  $\sqrt{s} = 13$  TeV (right). The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.

As three out of four azimuthal angles are always entering in (3.4), configurations with one jet recoiling against the other three are necessarily characterised by lower values of  $\Delta\varphi_{3j}^{min}$  with respect to the two-against-two topology; the minimum, in fact, will be obtained in the first case for  $i, j, k$  denoting the three jets in the same hemisphere, whereas no such a case is possible for the second configuration. Obviously, the first case would be allowed only by SPS in a collinear tree-level framework, whereas the second would be enhanced by DPS. In the  $k_T$ -factorization approach, this situation is smeared out by the presence of transverse momenta of the initial state partons. For our unintegrated parton distributions, the corresponding distributions are shown in Fig. 8. We do not see such obvious effects in the case of the  $k_T$ -factorization.

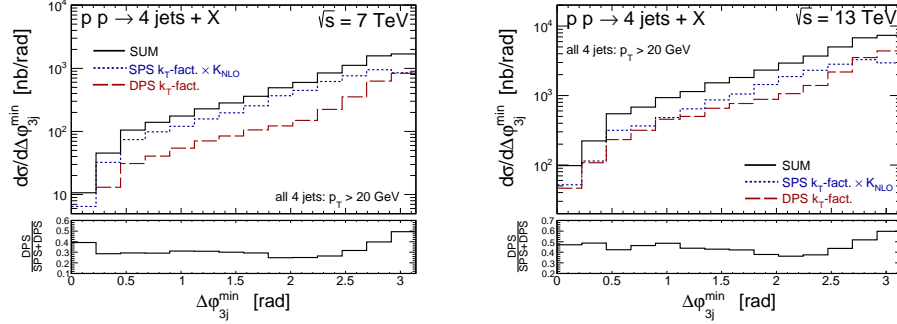


Figure 8: Distribution in  $\Delta\varphi_{3j}^{min}$  angle for the symmetric cut with  $p_T > 20$  GeV for  $\sqrt{s} = 7$  TeV (left) and  $\sqrt{s} = 13$  TeV (right). The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.

## 4 Conclusions

We have presented our recent results for four-jet production obtained for the first time within  $k_T$ -factorization approach. The calculation of the SPS contribution is a technical achievement. So far only production of the  $c\bar{c}c\bar{c}$  final state (also of the  $2 \rightarrow 4$  type) was discussed in the literature.

We have found that both collinear and the ( $k_T$ )-factorization approaches describe the data for hard central cuts, relevant for the ATLAS experiment, reasonably well when using the KMR TMDs. For the harder cuts we get both normalization and shape of the transverse momentum distributions. We nicely describe also CMS distribution for a special variable  $\Delta S$ .

In this presentation we have discussed also how to look at the DPS effects and how to maximize their role in four jet production. We found that, for sufficiently small cuts on the transverse momenta, DPS effects are enhanced relative to the SPS contribution: when rapidities of jets are large, for large rapidity distances between the most remote jets, for small azimuthal angles between the two jets most remote in rapidity and/or for large values of the  $\Delta\varphi_{3j}^{min}$  variable. For more details we refer the interested reader to our regular article [4].

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